Market Microstructure and Algorithmic Trading

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Stylised facts

Statistical properties of key variables in high frequency markets: **predictable behavior** of microstructure effects.

Examples:

- News imply peaks of volume and volatility (prices adjust).
- Activity on markets impact other markets (US markets lead the way).

U-shape of the traded volume:

- The effect of fixing auctions (more volume). Open: uncertainty on prices and unwind of overnight positions. Close: funds are benchmarked and market makers unwind.
- Influence from one market to another.

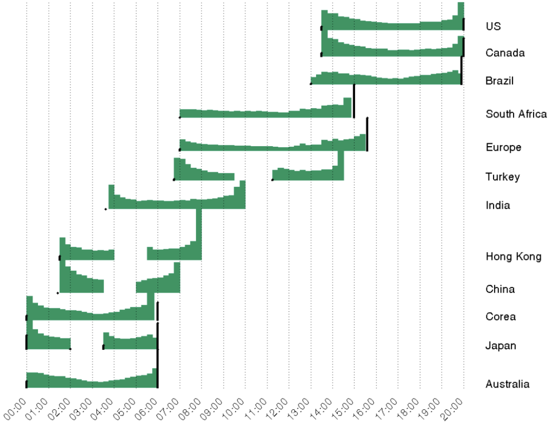


Figure: Trading volume distribution in the world. Source: (Lehalle and Laruelle 2018).

Bid-ask spread and volatility

- The more trades per day, the easiest to unwind inventory $\implies$ liquidity is cheaper. The more volatile the asset, the riskier it is to hold inventory.
- Universal relation between b-a spread η and volatility σ (see (Wyart et al. 2008)).

$$\eta \propto \frac{\sigma}{\sqrt{M}}, \quad M\text{: number of trades per day.}$$

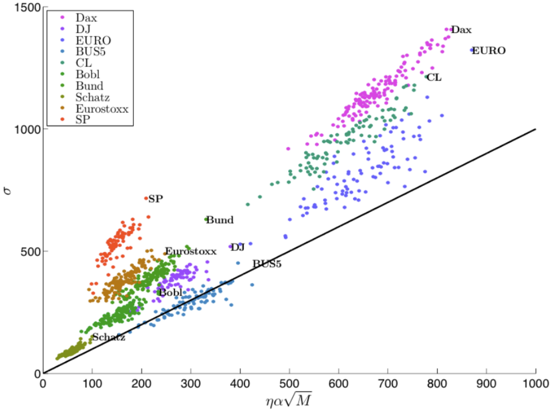


Figure: Source: (Lehalle and Laruelle 2018).

Volatility seasonality

- Volatility has seasonality: less intense at the end than at the start of the day.

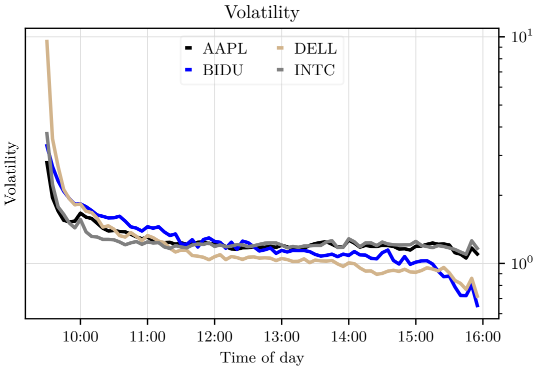


Figure: Average volatility throughout the trading day. Data is between January and March 2023

B.A-spread and volume on the book $\left(Q^{A}+Q^{B}\right)/2)$

- BA-spread is large at the start of the day, but finishes small because of market maker running to get rid of their inventory passively.
- VOB seasonality is the inverse of that of the B.A-spread.

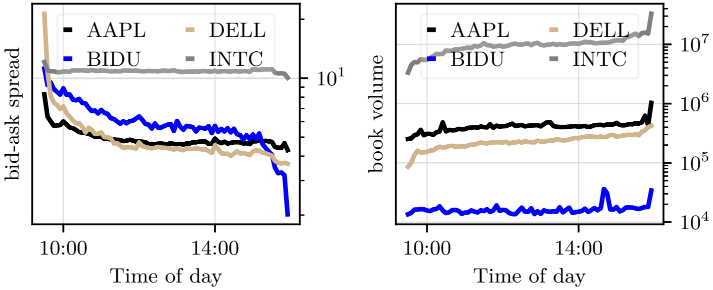
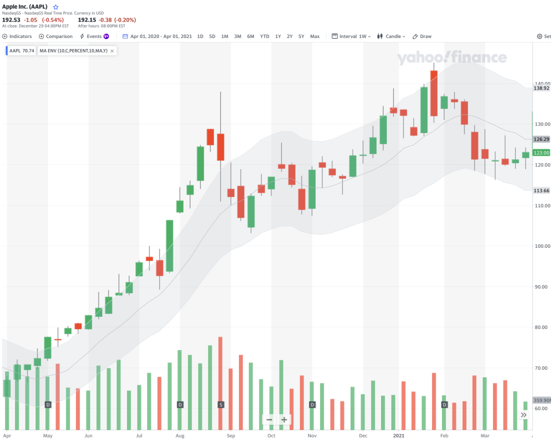


Figure: Average bid-ask spread and book volume throughout the trading day. Data is between January and March 2023

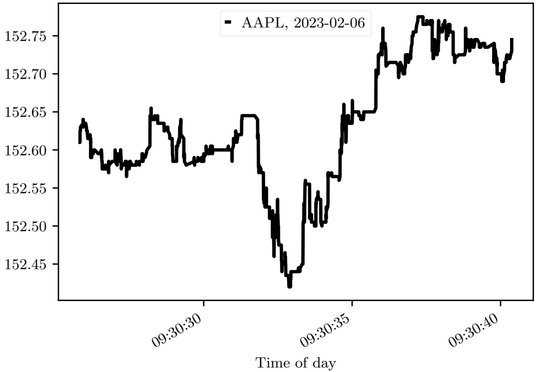
Asset prices transition through a number of **regimes**: momentum, mean reversion, and random walks.

They can be captured with technical indicators (RSI, ADX...).



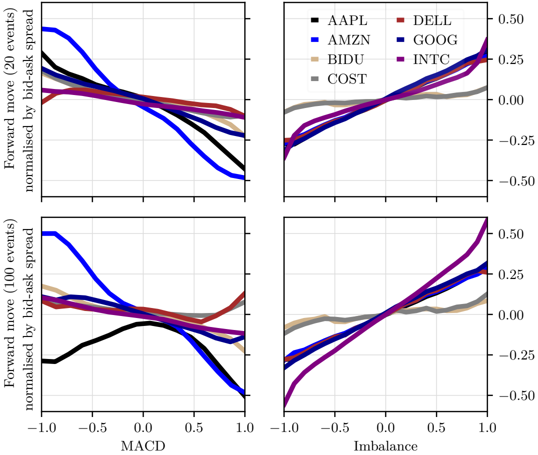
The regimes are observed at different time scales (low, medium, and high frequency).

The regimes depend on the asset class (more mean reversion in FX and Bonds markets).



At high frequency, market operators use directional signals.

Volume imbalance: $I_t^1 = \frac{Q_t^{\mathrm{B}} - Q_t^{\mathrm{A}}}{Q_t^{\mathrm{B}} + Q_t^{\mathrm{A}}} \in [-1, 1]$, MACD: $\begin{cases} \tilde{S}_t & = \mathrm{E}^{\varepsilon_1}(S_t) - \mathrm{E}^{\varepsilon_2}(S_t), \\ I_t^2 & = 10^5 \left(\tilde{S}_t - \mathrm{E}^{\varepsilon_3}\left(\tilde{S}_t\right)\right) / S_{t-\varepsilon_2-\varepsilon_3}, \end{cases}$



Left: Average price innovation after 20 events (top) and 100 events (bottom) normalised by the average bid-ask spread, as a function of MACD values. Right: Same for imbalance; data is between October 2022 and December 2022. Source: Cartea et al. 2023

Last but not least: assets are **cointegrated**, and they exhibit **lead-lag** relationships.

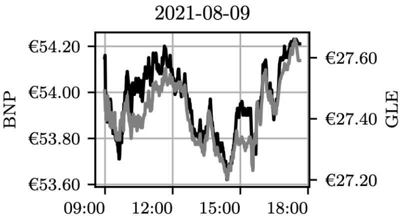


Figure: Mid-prices of BNP (left axis) and GLE (right axis) sampled every 60 seconds during the regular trading hours (09:00-17:30) over August 09 and August 10, 2021. Source: Bergault et al. 2022.

- An agent holds an **initial position** Q_0 at time $t = 0$ that they wish to **unwind** over a time window $[0, T]$.

- **Inventory**:

$$\boxed{dQ_t^\nu = \nu_t \, dt}$$

- **Price**:

$$\boxed{dS_t^\nu = \mu_t \, dt + k \, \nu_t \, dt + \sigma \, dW_t, \quad S_0^\nu = S_0,}$$

- **Cash**:

$$\boxed{dX_t^\nu = -\tilde{S}_t^\nu \, \nu_t \, dt = -\left(S_t^\nu + \eta \, \nu_t\right) \nu_t \, dt, \quad X_0^\nu = X_0.}$$

- **Admissible** strategies: **no unwind** constraint.

The performance criterion is the CJ type

$$H^{\nu}(t,x,s,\mu,q) = \mathbb{E}_{t,x,s,\mu,q}\left[X_T + Q^{\nu}_T\,(S^{\nu}_T - \alpha\,Q^{\nu}_T) - \phi\int_t^T (Q^{\nu}_u)^2\,du\right],$$

where $\mathbb{E}_{t,x,s,\mu,q}$ is the expectation conditioned on (with a slight abuse of notation) $X_t = x$, $S_t = S$, $\mu_t = \mu$, and $Q_t = q$.

The value function $H : [0,T] \times R^4 \mapsto \mathbb{R}$ of the agent is

$$H(t,x,s,\mu,q) = \sup_{\nu\in\mathcal{A}} H^{\nu}(t,x,s,\mu,q)\,,$$

What do the performance criterion terms represent ?

The dynamic programming principle for the value function suggests that H satisfies the HJB:

$$
\boxed{\begin{aligned}
0 = &\left(\partial_t + \tfrac{1}{2}\sigma^2\,\partial_{SS}\right) H + \mathcal{L}^{\mu} H - \phi\, q^2 && (1)\\
&+ \sup_{\nu} \left\{ \Big(-\nu\,(S + \eta\,\nu)\,\partial_x + (\mu + k\,\nu)\;\partial_S + \nu\,\partial_q \Big) H \right\}, && (2)
\end{aligned}}
$$

with terminal condition $H(T, x, s, \mu, q) = x + q\,s - \alpha\, q^2\,.$

Remember:

$$
\begin{cases}
dQ_t^{\nu} = & \nu_t\, dt\\
dS_t^{\nu} = & \mu_t\, dt + k\,\nu_t\, dt + \sigma\, dW_t\\
dX_t^{\nu} = & -\left(S_t^{\nu} + \eta\,\nu_t\right)\, \nu_t\, dt
\end{cases}
$$

Use the ansatz ...?

Use the ansatz ...?

$$\boxed{H(t,x,s,\mu,q) = x + q\,s + h(t,q,\mu)\,,}$$

and write

$$\begin{aligned} 0 =& \left(\partial_t + \tfrac{1}{2}\sigma^2\,\partial_{SS}\right) H + \mathcal{L}^\mu H - \phi\,q^2 \\ &+ \sup_\nu \left\{ \Big(-\nu\,(S + \eta\,\nu)\,\partial_x + (\mu + k\,\nu)\;\partial_S + \nu\,\partial_q \Big) H \right\}, \\ \implies\; 0 =& \left(\partial_t + \tfrac{1}{2}\sigma^2\,\partial_{SS}\right) h + \mathcal{L}^\mu h - \phi\,q^2 + \sup_\nu \left\{ \Big(-\eta\,\nu^2 + (\mu + k\,\nu)\;q \Big) h \right\} \end{aligned}$$

The HJB becomes

$$0 = \partial_t h + \mathcal{L}^\mu h - \phi\,q^2 + \mu\,q + \sup_\nu \Big\{ -\eta\,\nu^2 + \nu\;(k\,q + \partial_q h) \Big\}, \tag{3}$$

with terminal condition $h(T,q,\mu) = -\alpha\,q^2\,.$

Next step ?

Next step ? The optimal trading speed in **feedback form** is obtained by solving the first-order condition and we write

$$\boxed{\nu^\star = \frac{k\,q + \partial_q h}{2\,\eta}.} \tag{4}$$

Substitute to find

$$\Longrightarrow \quad \boxed{0 = \partial_t h + \mathcal{L}^\mu h - \phi\, q^2 + \mu\, q + \frac{(k\,q + \partial_q h)^2}{4\,\eta}.}$$

Next step ?

Next step ? Due to the existence of a linear and a quadratic term in q in and the form of the terminal condition of h, we use the ansatz

$$\boxed{h(t,\mu,q) = h_0(t,\mu) + q\,h_1(t,\mu) + q^2\,h_2(t,\mu)\,.}$$

Substitute to write

$$\begin{aligned}0 &= \partial_t h + \mathcal{L}^\mu h - \phi\,q^2 + \mu\,q + \frac{(k\,q + \partial_q h)^2}{4\,\eta}\\ &= (\partial_t + \mathcal{L}^\mu)\,h_0(t,\mu) + q\,(\partial_t + \mathcal{L}^\mu)\,h_1(t,\mu) + q^2\,(\partial_t + \mathcal{L}^\mu)\,h_2(t,\mu)\\ &\quad - \phi\,q^2 + \mu\,q + \frac{(k\,q + h_1(t,\mu) + 2\,q\,h_2(t,\mu))^2}{4\,\eta}\end{aligned}$$

collecting the terms in q, we find the ODE system

$$\boxed{\begin{cases}0 = & (\partial_t + \mathcal{L}^\mu)\,h_0 + \frac{1}{4\,\eta}h_1^2\\ 0 = & (\partial_t + \mathcal{L}^\mu)\,h_1 + \mu + \frac{1}{2\eta}\,h_1\,(k + 2h_2)\\ 0 = & (\partial_t + \mathcal{L}^\mu)\,h_2 - \phi + \frac{1}{4\eta}\,(k + 2h_2)^2\,,\end{cases}}$$

subject to terminal conditions $h_0\,(T,\mu) = h_1\,(T,\mu) = 0$ and $h_2(T,\mu) = -\alpha$.

The equation in h_2 contains no source terms in μ and its terminal condition does not depend on μ, thus the solution must be independent of μ and h_2 is only a function of time.

h_2 solves the (Riccati) ODE

$$0 = h_2'(t) - \phi + \frac{1}{4\eta}\,(\eta + 2\,h_2(t))^2\;,$$

which can be solved explicitly:

$$\boxed{h_2(t) = \sqrt{\eta\,\phi}\,\frac{1+\zeta\,e^{2\,\gamma\,(T-t)}}{1-\zeta\,e^{2\,\gamma\,(T-t)}} - \frac{1}{2}k\,,} \tag{5}$$

where

$$\gamma = \sqrt{\frac{\phi}{\eta}}, \quad \text{and} \quad \zeta = \frac{\alpha - \frac{k}{2} + \sqrt{\eta\,\phi}}{\alpha - \frac{k}{2} - \sqrt{\eta\,\phi}}.$$

Brief reminder: **Feynman-Kac** formula.

Assume you wish to solve

$$(\partial_t + \mathcal{L}^{\mu})\, C(t,\mu) = a\, C(t,\mu) + f(t,\mu)\,, \text{ with t.c. } C(T,\mu) = 0.$$

Use Ito's lemma to write

$$C(T,\mu_T) = C(t,\mu_t) + \int_t^T (\partial_t + \mathcal{L}^{\mu})\, C(s,\mu_s)\, ds.$$

Replace the term in $\partial_t + \mathcal{L}^{\mu}$ so we need to solve

$$\begin{aligned} C(T,\mu_T) &= C(t,\mu_t) + a\int_t^T C(s,\mu_s)\, ds + \int_t^T f(s,\mu_s)\, ds \\ \Longrightarrow C(t,\mu_t) &= c - \mathbb{E}_{t,\mu}\left[a\int_t^T C(s,\mu_s)\, ds + \int_t^T f(s,\mu_s)\, ds\right] \end{aligned}$$

Consider the candidate solution:

$$\hat{C}(t,\mu_t) = \mathbb{E}_{t,\mu}\left[-\int_t^T f(s,\mu_s)\, \exp(-a(s-t))\, ds\right].$$

Verify it is a solution, and by uniqueness it is the unique solution.

$$\begin{cases} 0 = & (\partial_t + \mathcal{L}^\mu)\, h_0 + \frac{1}{4\,\eta} h_1^2 \\ 0 = & (\partial_t + \mathcal{L}^\mu)\, h_1 + \mu + \frac{1}{2\eta}\, h_1\, (k + 2h_2) \\ 0 = & (\partial_t + \mathcal{L}^\mu)\, h_2 - \phi + \frac{1}{4\eta}\, (k + 2h_2)^2 \,, \end{cases}$$

Feynman-Kac for h_1 and h_0:

$$h_1(t,\mu) = \mathbb{E}_{t,\mu}\left[\int_t^T \exp\left\{\frac{1}{\eta}\int_t^u \left(h_2(s) + \tfrac{1}{2}k\right)\, ds\right\} \mu_u\, du\right] \,,$$

which is simplified to

$$h_1(t,\mu) = -\int_t^T \left(\frac{e^{-\gamma(T-u)} - \zeta e^{\gamma(T-u)}}{e^{-\gamma(T-t)} - \zeta e^{\gamma(T-t)}}\right) \mathbb{E}_{t,\mu}\left[\,\mu_u\,\right] du \,. \tag{6}$$

and

$$h_0(t,\mu) = \frac{1}{4\,\eta} \int_t^T \mathbb{E}_{t,\mu}\left[h_1^2(t,\mu)\right] .$$

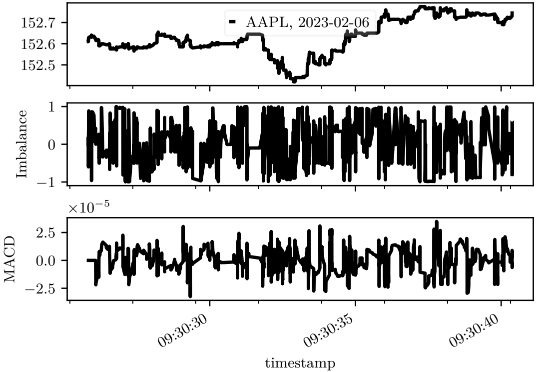
The optimal adaptive trading strategy is

$$\nu_t^* = \underbrace{-\gamma \frac{\zeta e^{\gamma(T-t)} + e^{-\gamma(T-t)}}{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}} Q_t^{\nu^*}}_{\text{Almgren-Chriss strategy}} + \underbrace{\frac{1}{2\eta} \int_t^T \left(\frac{\zeta e^{\gamma(T-u)} - e^{-\gamma(T-u)}}{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}} \right) \mathbb{E}\left[\mu_u \mid \mathcal{F}_t^{\mu}\right] du}_{\text{speculative component}}$$

The second term in the optimal strtegy is a speculative component:

- it adjusts the speed of trading using the weighted average of the future expected value of the signal μ over the remainder of the trading window
- the strategy gives more weight to signal values near t, and the contribution of the expected drift values decreases as it approaches the trading horizon (because of the terminal penalty).
- When the expected weighted drift is positive, the agent buys the asset. When the expected weighted drift is negative, the agent sells.
- The speculative component is significant when the ratio η is low.

Imbalance and MACD :



Imbalance and MACD signals for AAPL.

An agent wants to use Imbalance or MACD to execute better. They assume the mean reverting dynamics

$$d\mu_t = -\kappa\, \mu_t + \xi dB_t\,,$$

where κ drives the mean reversion speed to 0.

The solution to the SDE is

$$\mu_s = e^{-\kappa(s-t)}\mu_t + \int_t^s e^{-\kappa(s-t)}\xi\, dB_t,$$

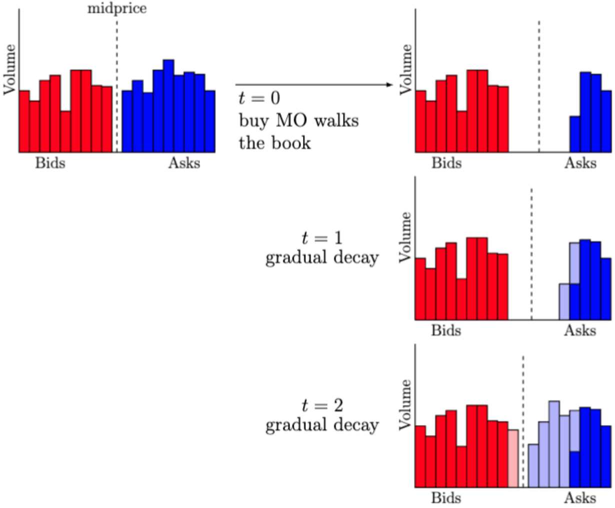
so the expected price drift is

$$\mathbb{E}\left[\mu_s \middle| \mathcal{F}_t^{\mu}\right] = e^{-\kappa(s-t)}\mu_t,$$

and the optimal trading speed is

$$\boxed{\nu_t^* = -\gamma\frac{\zeta e^{\gamma(T-t)} + e^{-\gamma(T-t)}}{\zeta e^{\gamma(T-t)} - e^{-\gamma(T-t)}} Q_t^{\nu^*} + \frac{1}{2\eta}\mu_t \int_t^T \frac{\zeta e^{2\gamma(T-u)} - 1}{\zeta e^{2\gamma(T-t)} - 1} e^{(\gamma-\kappa)(u-t)} du.}$$

Market impact in practice: resilience



In practice:

- The midprice gradually decays back to its original value.
- The rate at which orders are replensihed is called **resilience**.
- The frequency at which one sends large MOs is important.

What we assumed:

$$dS_t = k\,\nu_t + \sigma\,dW_t \tag{7}$$
$$dX_t = -\nu_t\,(S_t + \eta\,\nu_t)\ dt. \tag{8}$$

The assumption of permanent and temporary market impact implies

- Instantaneous resilience of the LOB.
- Instantaneous permanent impact on the price.

- An agent holds an **initial position** Q_0 at time $t = 0$ that they wish to **unwind** over a time window $[0, T]$.

- **Inventory**:

$$dQ_t^\nu = \nu_t \, dt$$

- **Price**:

$$S_t^\nu = S_0 + \underbrace{\sigma W_t}_{\text{market risk}} + \underbrace{k \int_0^t \nu_s ds}_{\text{permanent impact}} + \underbrace{\int_0^t h(\nu_s) \, G(t-s) \, ds}_{\text{transient impact}},$$

The function h determines the **magnitude** of **transient impact** and the **decay kernel** G controls how quickly the impact of trading decays through time.

- **Cash**:

$$dX_t^\nu = -\tilde{S}_t^\nu \, \nu_t \, dt = -\left(S_t^\nu + \eta \, \nu_t\right) \nu_t \, dt, \quad X_0^\nu = X_0.$$

The performance criterion

$$\boxed{H^{\nu}\left(t,x,s,,q\right)=\mathbb{E}_{t,x,s,q}\left[X_T+Q^{\nu}_T\left(S^{\nu}_T-\alpha\, Q^{\nu}_T\right)-\phi\int_t^T (Q^{\nu}_u)^2\,du\right]\,,}$$

where $\mathbb{E}_{t,x,s,q}$ is the expectation conditioned on (with a slight abuse of notation) $X_t=x$, $S_t=S$, and $Q_t=q$.

The value function $H:[0,T]\times R^3\mapsto\mathbb{R}$ of the agent is

$$H\left(t,x,s,\mu,q\right)=\sup_{\nu\in\mathcal{A}} H^{\nu}\left(t,x,s,\mu,q\right)\,,$$

- We consider the case of exponential decay and we write for $s \leq t$.

$$G(t-s) = \exp(-\beta(t-s))$$

The parameter ρ represents the rate of decay.

- We consider a linear instantaneous impact function

$$h : \nu \mapsto \lambda \, \nu.$$

The coefficient λ determines the distance between the midprices before and after an MO arrives in the LOB.

- The transient component of the midprice dynamics is now

$$\int_0^t \lambda \, \nu_s \, e^{-\beta\,(t-s)} \, ds.$$

- We define a new state variable I^ν, where $I_0 = 0$, that quantifies the accumulated transient impact:

$$I_t^\nu = \int_0^t \lambda \, \nu_s \, e^{-\beta\,(t-s)} \, ds \implies \boxed{dI_t^\nu = (\lambda \, \nu_t - \beta \, I_t^\nu) \; dt, \quad I_0 = 0.}$$

The dynamics of the price are

$$\boxed{dS_t^\nu = \sigma\, dW_t + \left(\tilde{\lambda}\,\nu_t - \beta\, I_t^\nu\right)\, dt, \quad S_0^\nu = S_0 \text{ known.}}$$

The performance criterion is

$$H^\nu\,(t,x,s,I,q) = \mathbb{E}_{t,x,s,I,q}\left[X_T + Q_T^\nu\,\left(S_T^\nu - \alpha\, Q_T^\nu\right) - \phi\,\int_t^T (Q_u^\nu)^2\, du\right],$$

The value function $H : [0, T] \times R^4 \mapsto \mathbb{R}$ of the agent is

$$H\,(t,x,s,I,q) = \sup_{\nu\in\mathcal{A}} H^\nu\,(t,x,s,I,q)\;.$$

The dynamcis of the problem are

$$
\begin{cases}
dQ_t^\nu &= \nu_t \, dt \\
dS_t^\nu &= \sigma \, dW_t + \left(\tilde{\lambda} \, \nu_t - \beta \, I_t^\nu\right) \, dt \\
dX_t^\nu &= -\left(S_t^\nu + \eta \, \nu_t\right) \, \nu_t \, dt \\
dI_t^\nu &= \left(\lambda \, \nu_t - \beta \, I_t^\nu\right) \, dt
\end{cases}
$$

The HJB is:

$$
\begin{aligned}
0 =& \partial_t H - \phi \, q^2 + \frac{1}{2}\sigma^2 \, \partial_{SS} H - \beta \, I \, \partial_S H - \beta \, I \, \partial_I H \\
&+ \sup_u \left\{ \tilde{\lambda} \, \nu \, \partial_S H + \lambda \, \nu \, \partial_I H + \nu \, \partial_q H - (S + \eta \, \nu) \, \nu \, \partial_x H \right\},
\end{aligned}
$$

subject to the terminal condition

$$
H(T, x, S, I, q) = x + q \, S - \alpha \, q^2.
$$

The usual steps

1. **Ansatz**

The usual steps

1. **Ansatz** $H(t, x, S, I, q) = x + q\,S + h(t, q, I)$:

$$\implies 0 = \partial_t h - \phi\, q^2 - \beta\, I\, q - \beta\, I\, \partial_I h + \sup_u \left\{ \nu\, \left(\tilde{\lambda}\, q + \lambda\, \partial_I h + \partial_q h \right) - \eta\, \nu^2 \right\},$$

subject to the terminal condition

$$h(T, I, q) = -\alpha\, q^2.$$

2.

The usual steps

1. **Ansatz** $H(t,x,S,I,q) = x + q\,S + h(t,q,I)$:

$$\implies 0 = \partial_t h - \phi\,q^2 - \beta\,I\,q - \beta\,I\,\partial_I h + \sup_u \left\{\nu\,\left(\tilde{\lambda}\,q + \lambda\,\partial_I h + \partial_q h\right) - \eta\,\nu^2\right\},$$

subject to the terminal condition

$$h(T,I,q) = -\alpha\,q^2.$$

2. The supremum term can be solved with a FOC. The optimal feedback speed:

$$\boxed{\nu^\star = \frac{\tilde{\lambda}\,q + \lambda\,\partial_I h + \partial_q h}{2\,\eta},}$$

The PDE simplifies to

$$0 = \partial_t h - \phi\,q^2 - \beta\,I\,q - \beta\,I\,\partial_I h + \frac{\left(\tilde{\lambda}\,q + \lambda\,\partial_I h + \partial_q h\right)^2}{4\,\eta}.$$

3. Linear and quadratic terms in q and I $\implies$

3. Linear and quadratic terms in q and $I \implies$ quadratic polynomial ansatz in q and I:

$$h(t,q,I) = A(t)q^2 + B(t)\,q\,I + C(t)\,I^2 + D(t)\,q + E(t)\,I + F(t)\,,$$

or equivalently

$$\boxed{h(t,q,I) = \begin{pmatrix} q \\ I \end{pmatrix}^{\intercal} P(t) \begin{pmatrix} q \\ I \end{pmatrix} + \begin{pmatrix} D(t) \\ E(t) \end{pmatrix}^{\intercal} \begin{pmatrix} q \\ I \end{pmatrix} + F(t)\,,}$$

where $P : [0, T] \to \mathcal{S}_2(\mathbb{R})$ is defined as

$$\boxed{P(t) = \begin{pmatrix} A(t) & \frac{1}{2}B(t) \\ \frac{1}{2}B(t)^{\intercal} & C(t) \end{pmatrix}.}$$

Some annoying algebra shows that for all q nad all I :

$$\begin{aligned}
0 = &\left(A'(t) - \phi + \frac{1}{4\,\eta}\left(\tilde{\lambda} + \lambda\,B(t) + 2\,A(t)\right)^2\right) q^2 \\
&+ \left(B'(t) - \beta - \beta\,B(t) + \frac{1}{2\,\eta}\left(\tilde{\lambda} + \lambda\,B(t) + 2\,A(t)\right)(2\,\lambda\,C(t) + B(t))\right) q\,I \\
&+ \left(C'(t) - 2\,\beta\,C(t) + \frac{1}{4\,\eta}(2\,\lambda\,C(t) + B(t))^2\right) I^2 \\
&+ \left(D'(t) + \frac{1}{2\,\eta}(\lambda\,E(t) + D(t))\left(\tilde{\lambda} + \lambda\,B(t) + 2\,A(t)\right)\right) q \\
&+ \left(E'(t) - \beta\,E(t) + \frac{1}{2\,\eta}(\lambda\,E(t) + D(t))(2\,\lambda\,C(t) + B(t))\right) I \\
&+ F'(t) + (\lambda\,E(t) + D(t))^2 .
\end{aligned}$$

We obtain the following system of ODEs

$$\begin{cases} 0 = & A'(t) - \phi + \frac{1}{4\eta}\left(\tilde{\lambda} + \lambda B(t) + 2A(t)\right)^2 \\ 0 = & B'(t) - \beta - \beta B(t) + \frac{1}{2\eta}\left(\tilde{\lambda} + \lambda B(t) + 2A(t)\right)(2\lambda C(t) + B(t)) \\ 0 = & C'(t) - 2\beta C(t) + \frac{1}{4\eta}(2\lambda C(t) + B(t))^2 \\ 0 = & D'(t) + \frac{1}{2\eta}(\lambda E(t) + D(t))\left(\tilde{\lambda} + \lambda B(t) + 2A(t)\right) \\ 0 = & E'(t) - \beta E(t) + \frac{1}{2\eta}(\lambda E(t) + D(t))(2\lambda C(t) + B(t)) \\ 0 = & F'(t) + (\lambda E(t) + D(t))^2 . \end{cases} \tag{9}$$

Notice that $D \equiv E \equiv 0$.

So $F \equiv 0$

The ODEs in A, B, C are a matrix Riccati :

$$\boxed{0 = P'(t) + Q + Y^{\intercal} P(t) + P(t) Y + P(t) U P(t),}$$

with terminal condition

$$P(T) = \begin{pmatrix} -\alpha & 0 \\ 0 & 0 \end{pmatrix},$$

where

$$\boxed{Q = \begin{pmatrix} -\phi + \frac{\tilde{\lambda}^2}{4\,\eta} & -\frac{\beta}{2} \\ -\frac{\beta}{2} & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} \frac{\tilde{\lambda}}{2\,\eta} & 0 \\ \frac{\tilde{\lambda}\,\lambda}{2\,\eta} & -\beta \end{pmatrix}, \quad \text{and } \; U = \frac{1}{\eta} \begin{pmatrix} 1 & \lambda \\ \lambda & \lambda^2 \end{pmatrix},}$$

- The optimal strategy is

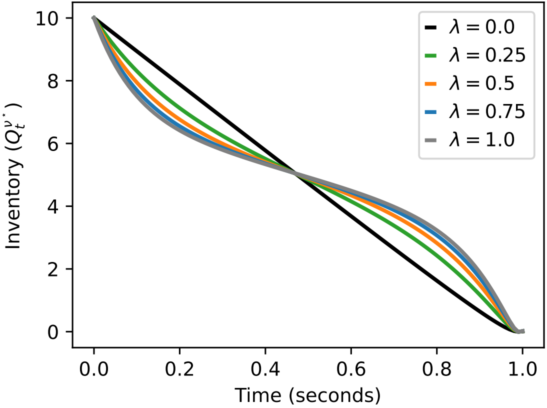
$$\boxed{\nu^\star = \frac{\tilde{\lambda}\, q + \lambda\, (B(t)\, q + 2\, C(t)\, I) + 2\, A(t) q + B(t)\, I}{2\, \eta}.}$$

- I^ν is deterministic, the strategy also is deterministic.

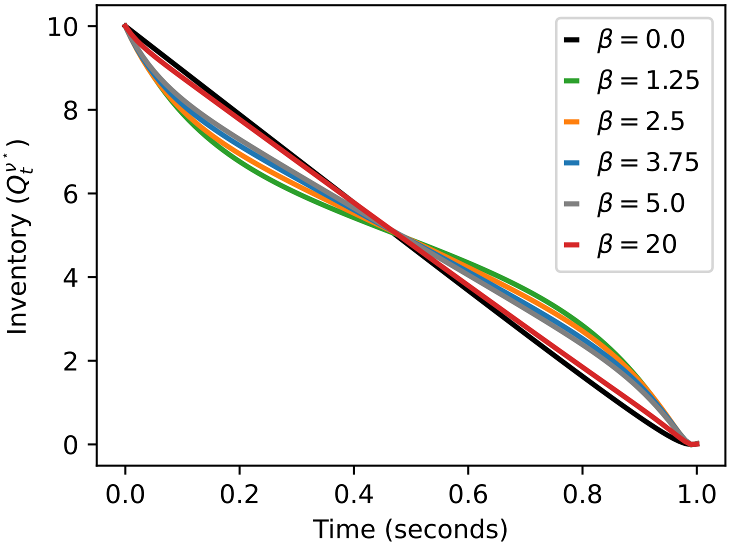
- The Riccati cannot be solved in closed-form. However, there exists very efficient approximation techniques.

We sole numerically the Riccati when: $T = 1$ second, $\eta = 0.01$, $\alpha = 10$, and $Q_0 = 10$.

We set a zero permanent impact ($k = 0$) and zero urgency ($\phi = 0$).



Optimal trading curve with transient impact price impact for various values of the transient impact parameter λ.



Optimal trading curve with transient impact price impact for various values of the decay coefficient β.

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