Market Microstructure and Algorithmic Trading

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- OTC markets are off-exchange “quote-driven” markets that are based on a network of market makers that set prices at which liquidity takers can trade.
- To set prices, liquidity providers in OTC markets constantly stream bid and ask quotes.
- They also respond to requests made by their clients: Request For Quotes.
- Market makers in OTC markets compete for clients.

Let's find a model for competition among market makers in OTC markets:

- Assume a market maker observes the midprice S_t of an asset and streams bid and ask quotes $S_t - \delta_t^b$ and $S_t + \delta_t^a$.
- Assume λ^b is the **intensity of the selling** trading flow that a market maker would receive if the price of liquidity δ^b at the bid is zero.
- Similarly, λ^a is the **baseline intensity of the buying trading flow**.
- The buy/sell trading flow is sensitive to the price of liquidity $\delta^{b,a}$:

$$\boxed{\mathbb{P}[\text{execution of receiving an order at the bid/ask}] = \exp\left\{-\kappa^{b,a}\,\delta\right\}}.$$

- Intensity of order arrival at the bid/ask for the market maker:

$$\boxed{\lambda^{b,a} \exp\left\{-\kappa^{b,a}\,\delta^{b,a}\right\}}.$$

Setup

- A market maker (MM) operates in an OTC market for a trading period $[0, T]$ (usually one trading day).
- They are in charge of streaming bid/ask quotes for a single asset whose midprice is modelled by the process S.
- The agent controls the depths δ^b and δ^a of the bid and ask quotes $S_t - \delta^b_t$ and $S_t + \delta^a_t$ that they propose around the midprice S.
- At every instant in the trading window $[0, T]$, the MM reassesses market conditions and the inventory level and streams new quotes $S_t - \delta^b_t$ and $S_t + \delta^a_t$.

Order arrival

- The baseline intensity of order arrival at the bid side is λ^b and at the ask side is λ^a.
- Two counting (controlled) processes $N_t^{b,\delta}$ and $N_t^{a,\delta}$ model the number of sell orders and buy orders, respectively, that the market maker fills.
- The orders arrive at Poisson times with intensities $\Lambda^b(\delta^b)$ and $\Lambda^a(\delta^a)$:

$$\boxed{\begin{cases} \Lambda^b(\delta^b) = & \lambda^b\, e^{-\kappa^b\, \delta^b} \\ \Lambda^a(\delta^a) = & \lambda^a\, e^{-\kappa^a\, \delta^a}\,, \end{cases}} \tag{1}$$

- $\kappa^b, \kappa^a > 0$ are the rates of decay of the sell and buy pressure, respectively, as a function of the price of liquidity.

Inventory

- The MM manages inventory risk: they choose boundaries on how long or short their position is in the security.
- Let Q denote the MM's inventory. The MM chooses $\underline{q} < 0$ and $\overline{q} > 0$ such that they fill trades only when the inventory satisfies $Q \in (\underline{q}, \overline{q})$.
- We modify the intensity functions above and we set

$$\boxed{\begin{cases} \Lambda^b(\delta^b) = & \lambda^b\, e^{-\kappa^b\, \delta^b}\, 1_{Q<\overline{q}} \\ \Lambda^a(\delta^a) = & \lambda^a\, e^{-\kappa^a\, \delta^a}\, 1_{Q>\underline{q}}\,, \end{cases}}$$

- The inventory Q and the boundaries $\underline{q}, \overline{q}$ are integers that represent a number of shares.

Dynamics

- **Price**:

$$\boxed{dSt = \sigma\, dW_t\,, \quad S_0 \in \mathbb{R}_+ \text{ is known,}}$$

- **Inventory**:

$$\boxed{dQ_t^\delta = dN_t^{\delta,b} - dN_t^{\delta,a}, \quad Q_0^\delta = 0.}$$

- **Cash**:

$$\boxed{dX_t^\delta = -\left(S_t - \delta_t^b\right)\, dN_t^{\delta,b} + \left(S_t + \delta_t^b\right)\, dN_t^{\delta,a}, \quad X_0^\delta = X_0 \in \mathbb{R} \text{ know.}}$$

The performance criterion

$$H^{\delta}(t,x,S,q) = \mathbb{E}_{t,\boldsymbol{x},q,S}\left[X_T + Q_T^{\delta}(S_T^{\delta} - \alpha Q_T^{\delta}) - \phi \int_{\boldsymbol{t}}^{T} (Q_u)^2 du\right],$$

where $\alpha \geq 0$ is the terminal penalty from liquidating any remaining inventory, $\phi \geq 0$ scales the quadratic running inventory penalty.

The value function

$$H(t,x,S,q) = \sup_{\delta \in \mathcal{A}} H^{\delta}(t,x,S,q),$$

where $\mathbb{E}_{t,x,S,q}$ is the expectation conditioned on (with a slight abuse of notation) $X_t = x$, $S_t = S$, and $Q_t = q$.

The dynamics are

$$\begin{cases} dSt & = \sigma\, dW_t \\ dQ_t^{\delta} & = dN_t^{\delta,b} - dN_t^{\delta,a} \\ dX_t^{\delta} & = -\left(S_t - \delta_t^b\right)\, dN_t^{\delta,b} + \left(S_t + \delta_t^b\right)\, dN_t^{\delta,a} \end{cases}$$

The HJB is

$$\boxed{\begin{aligned} 0 &= \partial_t H(t,x,q,S) + \frac{1}{2}\sigma^2\,\partial_{SS}H(t,x,q,S) - \phi\, q^2 && \text{(2a)} \\ &+ \sup_{\delta^a}\{\quad (H(\ ,\quad ,\quad ,\ ) - H(t,x,q,S))\}\ ? && \text{(2b)} \\ &+ \sup_{\delta^a}\{\quad (H(\ ,\quad ,\quad ,\ ) - H(t,x,q,S))\}\ ?, && \text{(2c)} \end{aligned}}$$

subject to the terminal condition

$$H(T,x,S,q) = x + q\,S - \alpha\, q^2.$$

The dynamics are

$$\begin{cases} dSt &= \sigma\, dW_t \\ dQ_t^{\delta} &= dN_t^{\delta,b} - dN_t^{\delta,a} \\ dX_t^{\delta} &= -\left(S_t - \delta_t^b\right)\, dN_t^{\delta,b} + \left(S_t + \delta_t^b\right)\, dN_t^{\delta,a} \end{cases}$$

The HJB is

$$0 = \partial_t H(t,x,q,S) + \frac{1}{2}\sigma^2\, \partial_{SS} H(t,x,q,S) - \phi\, q^2 \tag{3a}$$

$$+ \lambda^a \sup_{\delta^a} \left\{ e^{-\kappa^a\, \delta^a} \left(H(t, x + (S + \delta^a), q-1, S) - H(t,x,q,S) \right) \right\} 1_{q > \underline{q}} \tag{3b}$$

$$+ \lambda^b \sup_{\delta^b} \left\{ e^{-\kappa^b\, \delta^b} \left(H\left(t, x - (S - \delta^b), q+1, S\right) - H(t,x,q,S) \right) \right\} 1_{q < \bar{q}}, \tag{3c}$$

subject to the terminal condition

$$H(T, x, S, q) = x + q\, S - \alpha\, q^2.$$

Next step?

Next step? Propose the ansatz

$$\boxed{H(t,x,q,S) = x + q\,S + h(t,q).}$$

So

$$\begin{aligned}
H(t,x+(S+\delta^a),q-1,S) - H(t,x,q,S) =& x + (S+\delta^a) + (q-1)\;S + h(t,q-1) \\
& - x - q\,S - h(t,q) \\
=& \delta^a + h(t,q-1) - h(t,q) \\
H(t,x+(S-\delta^a),q+1,S) - H(t,x,q,S) =& \delta^b + h(t,q+1) - h(t,q)
\end{aligned}$$

Substitute in the HJB and find

$$0 = \partial_t h(t,q) - \phi q^2 + \lambda^a \sup_{\delta^a}\left\{e^{-\kappa^a\delta^a}\left(\delta^a + h(t,q-1) - h(t,q)\right)\right\} 1_{q>\underline{q}} \tag{4a}$$

$$+ \lambda^b \sup_{\delta^b}\left\{e^{-\kappa^b\delta^b}\left(\delta^b + h(t,q+1) - h(t,q)\right)\right\} 1_{q<\overline{q}}, \tag{4b}$$

Next step ?

Next step ? Solve the supremum to obtain the optimal feedback distances $\delta^{b,\star}$ and $\delta^{a,\star}$:

$$\delta^{b,*}(t,q) = \frac{1}{\kappa^b} - h(t,q+1) + h(t,q), \quad q \neq \overline{q}, \tag{5a}$$

$$\delta^{a,*}(t,q) = \frac{1}{\kappa^a} - h(t,q-1) + h(t,q), \quad q \neq \underline{q}. \tag{5b}$$

- The first components $1/\kappa^{a,b}$ optimise the instantaneous expected profit from a roundtrip trade: the expected profit from a roundtrip trade is

  $$\delta^b \, \Lambda^b(\delta^b) + \delta^a \, \Lambda^a(\delta^a)$$

  which is maximal for $\delta^b = 1/\kappa$ and $\delta^a = 1/\kappa$,

- The first component corresponds to the optimal strategy of a market maker who does not penalise inventory.

- The second component is related to changes in the value function due to an inventory change after trades are filled.

- **Assumption**: the intensity functions Λ^b and Λ^a decay at the same rate $\kappa = \kappa^a = \kappa^b$..

- We propose the following ansatz

$$\boxed{h(t,q) = \frac{1}{\kappa}\log\omega(t,q),}$$

- The HJB simplifies to

$$\begin{aligned}0 =& \partial_t\omega(t,q) - \phi\,\kappa\,q^2\,\omega(t,q)\\ &+ e^{-1}\lambda^a\,\omega\,(t,q-1)\,1_{q>\underline{q}} + e^{-1}\lambda^b\,\omega\,(t,q+1)\,1_{q<\overline{q}},\end{aligned}$$

subject to the terminal condition

$$\omega(T,q) = \exp(-\kappa\,\alpha\,q^2).$$

The equation

$$
\begin{aligned}
0 =& \partial_t \omega(t,q) - \phi\, \kappa\, q^2\, \omega(t,q) \\
&+ e^{-1} \lambda^a\, \omega\,(t, q-1)\, 1_{q>\underline{q}} + e^{-1} \lambda^b\, \omega\,(t, q+1)\, 1_{q<\overline{q}},
\end{aligned}
$$

is a system of ODEs:

- The inventory q can only take the values $\{\underline{q}, \underline{q}+1, \ldots, \overline{q}-1, \overline{q}\}$,
- For a fixed time t, $\omega(t,q)$ can only take the finitely many value $\{\omega(t,\underline{q}), \omega(t,\underline{q}+1), \ldots, \omega(t,\overline{q}-1), \omega(t,\overline{q})\}$.
- Define the vector $\mathbf{w}(t) = \big(\omega(t,\underline{q}), \omega(t,\underline{q}+1), \ldots, \omega(t,\overline{q}-1), \omega(t,\overline{q})\big)^{\mathsf{T}}$.
- $\mathbf{w}$ solves the ODE $\boxed{0 = \partial_t \mathbf{w}(t) + \mathbf{A}\, \omega(t),}$

$$
\text{where } \mathbf{A} \in \mathcal{M}_{\overline{q}-\underline{q}+1}(\mathbb{R}) \text{ and } \mathbf{A}_{i,q} = \begin{cases} -\phi\,\kappa\, q^2, & i = q, \\ \lambda^a\, e^{-1}, & i = q-1, \\ \lambda^b\, e^{-1}, & i = q+1, \\ 0, & \text{otherwise}, \end{cases}
$$

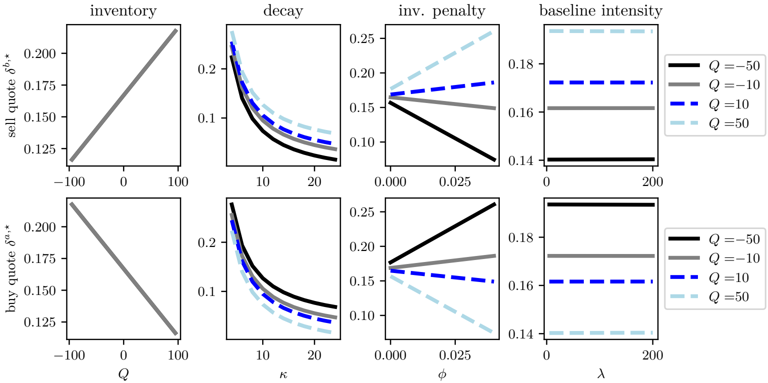
The solution to the first-order homogeneous matrix ODE is straightforward:

$$\boxed{\mathbf{w}(t) = \exp\left(\mathbf{A}\left(T - t\right)\right)\, \mathbf{z},} \tag{6}$$

where the vector $\mathbf{z}$ is $(\overline{q} - \underline{q} + 1)-$dimensional that writes

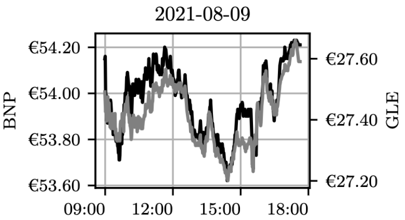
$$\mathbf{z} = \left(e^{-\alpha\,\kappa\,\underline{q}^2}, \ldots, e^{-\alpha\,\kappa\,\overline{q}^2}\right)^{\mathsf{T}}.$$

- Quotes as a function of inventory ?
- Quotes as a function of decay κ ?
- Quotes as a function of inv. penalty ϕ ?



Optimal distances $\delta^{b,\star}$ and $\delta^{a,\star}$ in as a function of model parameters for different values of the inventory. The default parameters are $\lambda = 50$, $T = 1$ minute, $\phi = 2 \times 10^{-3}$, $\kappa = 100$, and $Q_0 = 5$. The terminal time is $T = 30$ minutes and the terminal penalty is $\alpha = 10^{-4}$.

Assets are **cointegrated**, and they exhibit **lead-lag** relationships.



Mid-prices of BNP (left axis) and GLE (right axis) sampled every 60 seconds during the regular trading hours (09:00-17:30) over August 09 and August 10, 2021. Source: (Bergault et al. 2022).

When assets co-move together, it is key to

- Manage risk at the portfolio level.
- A position that is risky in one asset can be offset by another position.
- Liquidity should be managed at the portfolio level: impact in one asset leads to impact on another asset.

The new **FRTB** (Fundamental Review of the Trading Book) regulation will lead practitioners to assess liquidity risks within a **centralised risk book** for capital requirements.

We consider a market with $d \in \mathbb{N}^\star$ assets.

A classical model for cointegration is the multi-OU process

$$dS_t = R(\overline{S} - S_t)dt + VdW_t$$

- $\overline{S}$ represents the unconditional long-term expectation of $(S_t)_{t\in[0,T]}$.
- R steers the deterministic part of the process: mean reversion.
- V drives the dispersion. $\Sigma = VV^\intercal$ is the covariation matrix of the process.

OU processes are well suited when prices exhibit mean reversion and/or when there exist one or several linear combinations of asset prices that are stationary (cointegration).

Setup

- We consider a market with $d \in \mathbb{N}^\star$ assets.

- A trader wishes to liquidate their portfolio over a period of time $[0, T]$, with $T > 0$.

- The inventory process $Q = \left(Q_t^1, \ldots, Q_t^d\right)^\intercal$ evolves as

$$\boxed{dQ_t = v_t dt,} \quad Q_0 \in \mathbb{R}^d \text{ given}$$

- The fundamental prices of the d assets are a d-dimensional OU process $S = \left(S_t^1, \ldots, S_t^d\right)^\intercal$ with dynamics

$$\boxed{dS_t = R(\overline{S} - S_t)dt + VdW_t, \quad S_0 \in \mathbb{R}^d \text{ given}}$$

where $R \in \mathcal{M}_d(\mathbb{R})$, $V \in \mathcal{M}_{d,k}(\mathbb{R})$.

- The price is linearly impacted. We define the market price $\tilde{S}_t = \left(\tilde{S}_t^1, \ldots, \tilde{S}_t^d\right)^\intercal$ with dynamics:

$$d\tilde{S}_t = dS_t + Kv_t dt,$$

where $K \in \mathcal{S}_d(\mathbb{R})$ is the permanent impact matrix.

- $\tilde{X}_t$ is the trader's cash account

$$d\tilde{X}_t = -v_t^\intercal \tilde{S}_t dt - v^\intercal \eta\, v\, dt,$$

where $\eta \in \mathcal{S}_d^{++}(\mathbb{R})$ is the cross-impact matrix.

The performance criterion

$$\sup_{v\in\mathcal{A}} \mathbb{E}\left[-\exp\left(-\gamma\left(\tilde{X}_T + Q_T^\intercal \tilde{S}_T - Q_T^\intercal \tilde{\Gamma}\, Q_T\right)\right)\right],$$

where $\tilde{\Gamma} \in \mathcal{S}_d^+(\mathbb{R})$.

An equivalent problem if we assue $\Gamma = \tilde{\Gamma} - \frac{1}{2}K \in \mathcal{S}_d^+(\mathbb{R})$

$$\begin{aligned}
\tilde{X}_T + Q_T^\intercal \tilde{S}_T - Q_T^\intercal \tilde{\Gamma}\, Q_T &= \tilde{X}_0 + Q_0^\intercal \tilde{S}_0 + \int_0^T Q_t^\intercal d\tilde{S}_t - \int_0^T v^\intercal \eta\, v\, dt - Q_T^\intercal \tilde{\Gamma}\, Q_T \\
&= X_0 + Q_0^\intercal S_0 + \int_0^T Q_t^\intercal dS_t + \int_0^T Q_t^\intercal K v_t dt \\
&\quad - \int_0^T v^\intercal \eta\, v\, dt - Q_T^\intercal \tilde{\Gamma}\, Q_T \\
&= X_T + Q_T^\intercal S_T - Q_T^\intercal \left(\tilde{\Gamma} - \frac{1}{2}K\right) Q_T - \frac{1}{2} Q_0^\intercal K Q_0,
\end{aligned}$$

where

$$\boxed{dX_t = -v_t^\intercal S_t dt - v^\intercal \eta\, v\, dt.}$$

Performance criterion is now

$$\sup_{v\in\mathcal{A}} \mathbb{E}_{t,x,q,s}\left[-\exp\left(-\gamma\left(X_T + Q_T^{\intercal} S_T - Q_T\,\Gamma\,Q_T\right)\right)\right].$$

The value function of the problem $u : [0, T] \times \mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}$ is

$$u(t, x, q, s) = \sup_{v\in\mathcal{A}_t} \mathbb{E}_{t,x,q,s}\left[-\exp\left(-\gamma\left(X_T + (Q_T)^{\intercal} S_T - Q_T\,\Gamma\,Q_T\right)\right)\right].$$

The dynamics are

$$\begin{cases} dQ_t = & v_t dt, \\ dS_t = & R(\overline{S} - S_t)dt + VdW_t, \\ dX_t = & -v_t^\intercal S_t dt - v_t^\intercal \, \eta \, v_t \, dt \end{cases}$$

The HJB is

$$\boxed{\begin{aligned} 0 = \partial_t w + \sup_{v \in \mathbb{R}^d} & \left(-(v^\intercal S + L(v))\partial_x w + v^\intercal \nabla_q w\right) \\ & + (\overline{S} - S)^\intercal R^\intercal \nabla_S w + \frac{1}{2} \mathrm{Tr}\left(\Sigma D^2_{SS} w\right), \end{aligned}} \tag{7a}$$

with the terminal condition

$$w(T, x, q, s) = -\exp\left(-\gamma\left(x + q^\intercal s - q^\intercal \,\Gamma\, q\right)\right) \quad \forall (x, q, s) \in \mathbb{R} \times \mathbb{R}^d \times \mathbb{R}^d.$$

Next step ?

Next step ? Use the ansatz

$$w(t,x,q,s) = -\exp\left(-\gamma\left(x + q^\intercal s + \theta(t,q,s)\right)\right), \tag{8a}$$

$$\forall (t,x,q,s) \in [0,T] \times \mathbb{R} \times \mathbb{R}^{2d}, \tag{8b}$$

which simplifies the HJB to find

$$0 = \partial_t \theta + \sup_{v \in \mathbb{R}^d} \left(v^\intercal \nabla_q \theta - L(v)\right) + \frac{1}{2}\mathrm{Tr}\left(\Sigma D^2_{SS}\theta\right) - \frac{\gamma}{2}(q + \nabla_S \theta)^\intercal \Sigma (q + \nabla_S \theta) + (\overline{S} - S)^\intercal R^\intercal (\nabla_S \theta + q)$$

with terminal condition

$$\theta(T,q,s) = -q^\intercal \, \Gamma \, q \quad \forall (q,s) \in \mathbb{R}^d \times \mathbb{R}^d,$$

Next step ?

Next step ? Solve the sup term

$$
\boxed{\begin{aligned} 0 =& \partial_t\theta + \frac{1}{4}\nabla_q\theta^\intercal\eta^{-1}\nabla_q\theta + \frac{1}{2}\mathrm{Tr}\left(\Sigma D^2_{SS}\theta\right) \\ &- \frac{\gamma}{2}(q + \nabla_S\theta)^\intercal\Sigma(q + \nabla_S\theta) + (\overline{S} - S)^\intercal R^\intercal(\nabla_S\theta + q), \end{aligned}} \tag{9a}
$$

Next step ?

Next step ? Solve the sup term

$$\boxed{\begin{aligned} 0 =&\partial_t\theta + \frac{1}{4}\nabla_q\theta^\intercal\eta^{-1}\nabla_q\theta + \frac{1}{2}\mathrm{Tr}\left(\Sigma D^2_{SS}\theta\right) \\ &- \frac{\gamma}{2}(q+\nabla_S\theta)^\intercal\Sigma(q+\nabla_S\theta) + (\overline{S}-S)^\intercal R^\intercal(\nabla_S\theta + q), \end{aligned}} \tag{9a}$$

Next step ? Use the ansatz

$$\theta(t,q,s) = q^\intercal A(t)q + q^\intercal B(t)s + s^\intercal C(t)s + D(t)^\intercal q + E(t)^\intercal s + F(t)$$

or equivalently

$$\boxed{\theta(t,q,s) = \begin{pmatrix} q \\ s \end{pmatrix}^\intercal P(t) \begin{pmatrix} q \\ s \end{pmatrix} + \begin{pmatrix} D(t) \\ E(t) \end{pmatrix}^\intercal \begin{pmatrix} q \\ s \end{pmatrix} + F(t)\,,}$$

where

$$\boxed{P(t) = \begin{pmatrix} A(t) & \frac{1}{2}B(t) \\ \frac{1}{2}B(t)^\intercal & C(t) \end{pmatrix}.}$$

Many calculations later, we find that D, E, and F satisfy linear ODEs $\implies$ a solution exists.

$P = \begin{pmatrix} A & \frac{1}{2}B \\ \frac{1}{2}B & C \end{pmatrix}$ satisfies the Matrix Riccati ODE:

$$\boxed{P'(t) = Q + Y^\intercal P(t) + P(t)Y + P(t)UP(t),}$$

where

$$Q = \frac{1}{2}\begin{pmatrix} \gamma\Sigma & R \\ R^\intercal & 0 \end{pmatrix} \in \mathcal{S}_{2d}(\mathbb{R}), \qquad Y = \begin{pmatrix} 0 & 0 \\ \gamma\Sigma & R \end{pmatrix} \in \mathcal{M}_{2d}(\mathbb{R}),$$

$$U = \begin{pmatrix} -\eta^{-1} & 0 \\ 0 & 2\gamma\Sigma \end{pmatrix} \in \mathcal{S}_{2d}(\mathbb{R}),$$

and the terminal condition is

$$P(T) = \begin{pmatrix} -\Gamma & 0 \\ 0 & 0 \end{pmatrix} \in \mathcal{S}_{2d}(\mathbb{R}).$$

Mean-reverting asset

FX futures contract CDU1 on the currency pair Canadian Dollar (CAD) / US Dollar (USD) that is exchanged on the Chicago Mercantile Exchange.

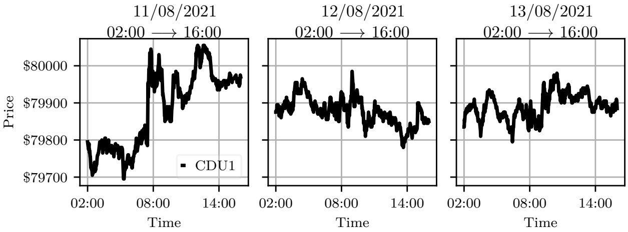


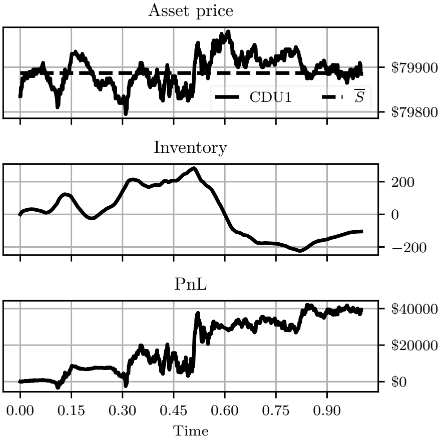
Figure: Mid-price of CDU1 sampled every 60 seconds during the regular trading hours (02:00-16:00 CT). Top left: August 11, 2021. Top right: August 12, 2021. Bottom: August 13, 2021.

A trader wishes to profit from mean reversion.

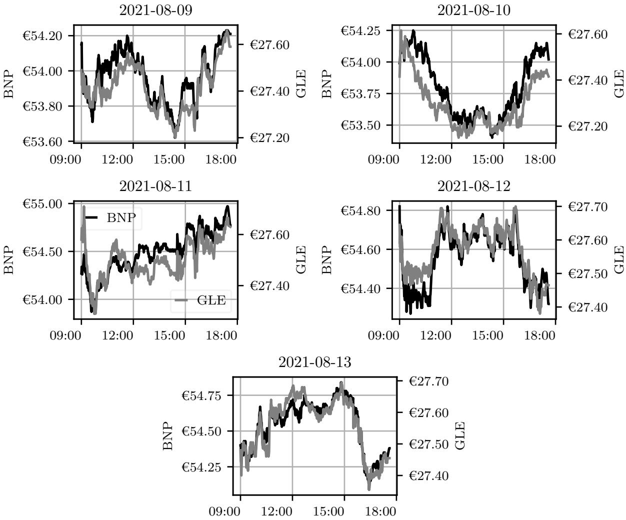
A time discretisation of an OU is an Auto-Regressive model of order 1, or $\mathrm{AR}(1)$. The parameters are estimated with a least squares regression using data of the two first days.

Parameter	Value
T	$1\ \mathrm{day}$
Q_0	0
S_0	\$79835
R	$5.1\ \mathrm{day}^{-1}$
$\overline{S}$	\$79887
σ	$243.67\ \$ \cdot \mathrm{day}^{-\frac{1}{2}}$
η	$5 \cdot 10^{-3}\ \$ \cdot \mathrm{day}$
Γ	\$100
γ	$2 \cdot 10^{-5}\ \$^{-1}$

Table: Value of the parameters.



Top: CDU1 price trajectory on August 13, 2021 – $(S_t)_{t\in[0,T]}$. Middle: Trajectory of the inventory when using the optimal strategy starting from $Q_0 = 0$ – $(Q_t)_{t\in[0,T]}$. Bottom: Corresponding trajectory of the PnL – $(X_t + Q_t S_t)_{t\in[0,T]}$.



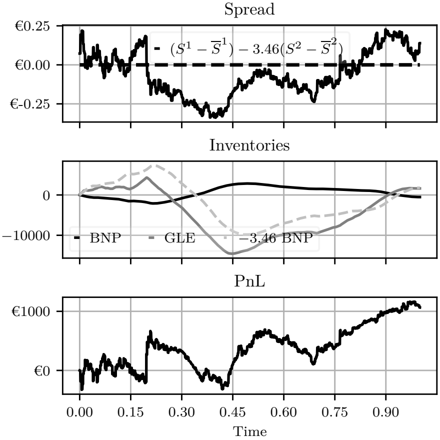
Mid-prices of BNP (left axis) and GLE (right axis) sampled every 60 seconds during the regular trading hours (09:00-17:30) over the week August 09-August 13, 2021.

The space of cointegration vectors is spanned by $(1, -3.46)$.

MOU in discrete-time is a VAR(1) model. We use least-squares again:

Parameter	Estimate
R	$\begin{pmatrix} 0.33 & 3.95 \\ -2.52 & 10.23 \end{pmatrix} \mathrm{day}^{-1}$
$\overline{S}$	$\left(\overline{S}^1, \overline{S}^2\right) = (54.23, 27.45)$
Σ	$\begin{pmatrix} 0.47 & 0.20 \\ 0.20 & 0.14 \end{pmatrix}^2 \cdot \mathrm{day}^{-1}$

Table: Multi-OU estimated parameters for the pair (BNP, GLE).



Top: Trajectory of the spread on August 13, 2021 – $((S^1_t - \overline{S}^1) - 3.46(S^2_t - \overline{S}^2))_{t\in[0,T]}$. Middle: Trajectory of the inventories when using the optimal strategy corresponding to the estimated multi-OU process with $\gamma = 2 \cdot 10^{-3}\ ^{-1}$ – $(Q_t)_{t\in[0,T]}$. Bottom: Trajectory of the PnL – $(X_t + Q_t^\intercal S_t)_{t\in[0,T]}$.

Bergault, Philippe, Fayçal Drissi, and Olivier Guéant (2022). “Multi-asset Optimal Execution and Statistical Arbitrage Strategies under Ornstein–Uhlenbe Dynamics”. In: **SIAM Journal on Financial Mathematics** 13.1, pp. 353–390.